

# Tables of Ramanujan series with rational values of $z$

There are 36 Ramanujan series corresponding to rational values of the parameter  $z$ . They can be written in the form

$$\sum_{n=0}^{\infty} z^n \frac{\left(\frac{1}{2}\right)_n (s)_n (1-s)_n}{(1)_n^3} (a+bn) = \frac{c}{\pi},$$

where  $a$  and  $b$  are positive integers and  $c^2$  is a rational number. We show them in the following tables (we are not completely sure of not having omitted someone). In addition, we have marked with a X those that have been proved until the date by the WZ-method (see [1], [2], [3], [4] and [5]).

Case  $s = 1/2$ :

$z$	$a$	$b$	$c$	WZ	$z$	$a$	$b$	$c$	WZ
-1	1	4	2	X	1/4	1	6	4	X
-1/8	1	6	$2\sqrt{2}$	X	1/64	5	42	16	X

Case  $s = 1/4$ :

$z$	$a$	$b$	$c$	WZ	$z$	$a$	$b$	$c$	WZ
-1/4	3	20	8	X	32/81	1	7	9/2	X
$-(16/63)^2$	8	65	$9\sqrt{7}$		1/9	1	8	$2\sqrt{3}$	
-1/48	3	28	$16/\sqrt{3}$	X	1/81	1	10	$9\sqrt{2}/4$	
-1/18 <sup>2</sup>	23	260	72		1/49 <sup>2</sup>	3	40	$49\sqrt{3}/9$	
-1/(72 <sup>2</sup> · 5)	41	644	$288/\sqrt{5}$		1/99 <sup>2</sup>	19	280	$18\sqrt{11}$	
-1/882 <sup>2</sup>	1123	21460	3528		1/99 <sup>4</sup>	1103	26390	$9801\sqrt{2}/4$	

Case  $s = 1/3$ :

$z$	$a$	$b$	$c$	$z$	WZ	$a$	$b$	$c$	WZ
-9/16	1	5	$4\sqrt{3}$	X	1/2	1	6	$3\sqrt{3}$	X
-1/16	7	51	$12\sqrt{3}$	X	2/27	2	15	27/4	
-1/80	5	45	$4\sqrt{15}$		4/125	4	33	$15\sqrt{3}/2$	
-1/2 <sup>10</sup>	53	615	$96\sqrt{3}$						
-1/3024	91	1155	$108\sqrt{7}$						
-1/500 <sup>2</sup>	827	14151	$1500\sqrt{3}$						

Case  $s = 1/6$ :

$z$	$a$	$b$	$c$	WZ	$z$	$a$	$b$	$c$	WZ
$-(4/5)^3$	8	63	$5\sqrt{15}$		$(3/5)^3$	3	28	$5\sqrt{5}$	
$-(3/8)^3$	15	154	$32\sqrt{2}$	X	$4/5^3$	2	22	$5\sqrt{15}/3$	
$-1/8^3$	25	342	$32\sqrt{6}$		$(2/11)^3$	20	256	$\sqrt{33}/11$	
$-9/40^3$	31	506	$160\sqrt{30}/9$		$(4/85)^3$	144	2394	$\sqrt{85^3}/\sqrt{3}$	
$-1/80^3$	263	5418	$1/2 \cdot 80^2/\sqrt{15}$						
$-1/440^3$	10177	261702	$3 \cdot 440^2/\sqrt{330}$						
$-1/53360^3$	13591409	545140134	$3/2 \cdot 53360^3/\sqrt{10005}$						

## References

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- [2] Guillera, J.: Some binomial series obtained by the WZ-method. Advances in Applied Mathematics **29**, 599-603, (2002).
- [3] Guillera, J.: Generators of Some Ramanujan Formulas. The Ramanujan Journal **11**, 41-48, (2006).
- [4] Guillera, J.: Series de Ramanujan: Generalizaciones y conjeturas. Thesis defended on July-2, 2007 at the University of Zaragoza (Spain).
- [5] Guillera, J.: About the WZ-pairs which prove Ramanujan series. Submitted for publication