

MY FORMULAS FOR $1/\pi^2$

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Identities 1 and 2

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n^5}{n!^5} = \frac{1}{2^{10n}} \frac{(2n)!^5}{n!^{10}},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{20n^2 + 8n + 1}{2^{2n}} = \frac{8}{\pi^2}, \quad (1)$$

and

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{820n^2 + 180n + 13}{2^{10n}} = \frac{128}{\pi^2}, \quad (2)$$

these identities have been proved by me.

Identity 3

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n^3 \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n}{n!^5} = \frac{1}{2^{12n}} \frac{(2n)!^2 (4n)!}{n!^8},$$

then

$$\sum_{n=0}^{\infty} A_n \frac{120n^2 + 34n + 3}{2^{4n}} = \frac{32}{\pi^2}. \quad (3)$$

this identity has been proved by me.

Identity 4

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n \left(\frac{1}{3}\right)_n \left(\frac{2}{3}\right)_n}{n!^5} = \frac{1}{2^{8n} 3^{3n}} \frac{(3n)!(4n)!}{n!^7},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{252n^2 + 63n + 5}{48^n} = \frac{48}{\pi^2}, \quad (4)$$

this identity remains unproved.

Identity 5

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n \left(\frac{1}{6}\right)_n \left(\frac{5}{6}\right)_n}{n!^5} = \frac{1}{2^{12n} 3^{3n}} \frac{(4n)!(6n)!}{(2n)!(3n)!n!^5},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{1640n^2 + 278n + 15}{2^{10n}} = \frac{256\sqrt{3}}{3\pi^2}, \quad (5)$$

this identity remains unproved.

Identity 6

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{8}\right)_n \left(\frac{3}{8}\right)_n \left(\frac{5}{8}\right)_n \left(\frac{7}{8}\right)_n}{n!^5} = \frac{1}{2^{18n}} \frac{(2n)!(8n)!}{(4n)!n!^6},$$

then

$$\sum_{n=0}^{\infty} A_n \frac{1920n^2 + 304n + 15}{7^{4n}} = \frac{56\sqrt{7}}{\pi^2}, \quad (6)$$

this identity remains unproved.

Identity 7

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{3}\right)_n \left(\frac{2}{3}\right)_n \left(\frac{1}{6}\right)_n \left(\frac{5}{6}\right)_n}{n!^5} = \frac{1}{36^{3n}} \frac{(6n)!}{n!^6},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{5418n^2 + 693n + 29}{80^{3n}} = \frac{128\sqrt{5}}{\pi^2}, \quad (7)$$

this identity remains unproved.

Identity 8

Let

$$A_n = \binom{2n}{n}^2 \sum_{j=0}^n \binom{2n-2j}{n-j}^2 \binom{2j}{j}^2,$$

then

$$\sum_{n=0}^{\infty} A_n \frac{36n^2 + 12n + 1}{2^{10n}} = \frac{32}{\pi^2}, \quad (8)$$

this identity remains unproved.